

# A Branch-Line Coupler with Two Arbitrary Operating Frequencies Using Left-Handed Transmission Lines

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**Abstract** — A branch-line coupler (BLC) working at two arbitrary frequencies using left-handed (LH) transmission lines (TLs) is presented. The analysis of the structure is based on the even-odd mode analysis of the conventional BLC as well as a recently developed model for the LH-TL. We demonstrate that the two operating frequencies can be obtained by tuning the phase slope of the different line sections. A prototype is demonstrated by both simulation and measurement results. The operating frequencies of the two pass-bands are 920 MHz and 1740 MHz, respectively.

## I. INTRODUCTION

Recently, the LH materials (LHM) have gained significant interest in the microwave community [1]. Smith *et al.* and Pendry *et al.* demonstrated practical realizations of the LHM [2-5]. Eleftheriades *et al.* proposed a lumped-element (LE) approach of a two-dimensional structure [6]. A theory and implementation of a LH-TL was recently introduced by Caloz *et al* [7]. The equivalent LE model of the LH-TL shows that it provides negative phase delay or phase advance. On the other hand, the conventional TL, which is referred to as the right-handed (RH) TL (RH-TL) in this paper, has positive phase delay. Based on previous analytical studies of the LHM several practical applications and usages in the design of couplers and antennas have been proposed [8].

The conventional BLC is made up of quarter wavelength lines; it can only operate at the fundamental frequency and at its odd harmonics [9], [10]. For use in modern wireless communication standards with two frequency bands, dual band components are required in order to reduce number of components in such a system.

In this paper, the LH-TL concept is applied to realize a versatile design of the BLC in which the second operating frequency can be arbitrarily designed. The negative phase delay extends the flexibility of the phase control of each branch-line in the BLC. Thus, the design proposed in this paper provides a way for using one single quadrature hybrid for two arbitrary frequencies.

## II. LEFT-HANDED TRANSMISSION LINES (LH-TL) AND COMPOSITE RIGHT/LEFT-HANDED TL (CRLH-TL)

The unit-cell of the artificial LE implementation of the RH and LH TLs are shown in Fig.1 [7]. It is obtained by cascading  $N$  times the unit-cells shown in Fig. 1, provided that the phase-shift induced by these unit-cells be much smaller than  $2\pi$ .

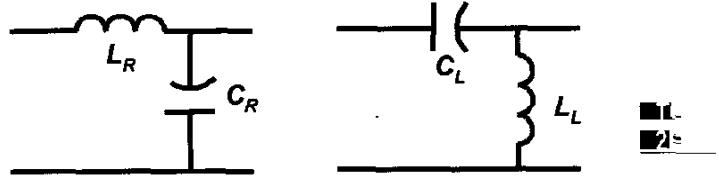


Fig. 1. Unit-cells of artificial RH and LH TLs, respectively. (a) RH-TL. (b) LH-TL.

The LH-TL is the electrical dual of the conventional RH-TL, in which the inductance and capacitance have been interchanged. The phase delay of the unit-cell of the artificial RH and LH-TL are

$$\phi_R = -\arctan \left[ \frac{\omega(L_R/Z_{0R} + C_R Z_{0R})}{(2 - \omega^2 L_R C_R)} \right] < 0, \quad (1a)$$

$$\phi_L = -\arctan \left[ \frac{\omega(L_L/Z_{0L} + C_L Z_{0L})}{(1 - 2\omega^2 L_L C_L)} \right] > 0, \quad (1b)$$

with the characteristic impedances

$$Z_{0R} = \sqrt{L_R/C_R}, \quad Z_{0L} = \sqrt{L_L/C_L}, \quad (2)$$

where the indices  $R$  and  $L$  refer to RH and LH, respectively. The RH-LH has a negative phase (phase lag), while the LH-TL has a positive phase (phase advance).

A CRLH-TL is the series combination of a LH-TL and a RH-TL, leading to the phase delay of a unit-cell of the artificial CRLH-TL

$$\phi_C = \phi_R + \phi_L, \quad (3)$$

where index  $C$  denotes CRLH, which becomes  $N\phi_C$  for the  $N$ -cells implementation of the line. At low frequencies, the

phase response is dominated by the LH contribution; at high frequencies, the phase response is dominated by the RH contribution.

The phase response of a CRLH-TL is shown in Fig. 2. The LH-TL provides an offset from DC in the lower frequency range, while the RH-TL provides an arbitrary slope in the upper frequency range, which is the range of operation for the BLC proposed in this paper. The combination of these two effects allows the possibility to reach any desired pair of frequencies. This is in contrast to the conventional case where, once the operating frequency corresponding to  $90^\circ$  is chosen, the next usable frequency necessarily corresponds to  $270^\circ$  because the phase curve is a straight line from DC to that frequency.

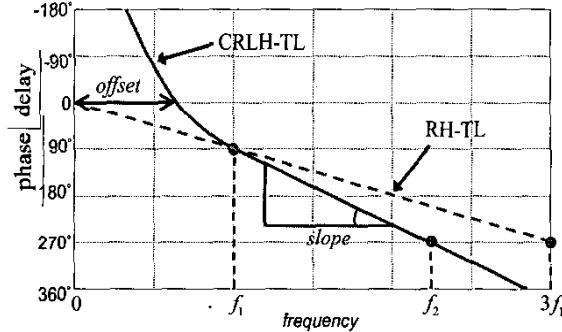


Fig. 2. Typical phase response of the RH- and the CRLH-TL.

### III. PRINCIPLE OF ARBITRARY FREQUENCY OPERATION

Each branch-line of the proposed coupler is designed as a CRLH-TL. The two  $Z_0$  lines have a characteristic impedance of  $50\ \Omega$  and the two  $Z_0/\sqrt{2}$  lines have the characteristic impedance of  $35\ \Omega$ . If the operating frequencies are chosen as  $f_1$  and  $f_2$  in Fig. 2, the phase delays of the CRLH-TL at  $f_1$  and  $f_2$  can be written

$$N\phi_c(f_1) = \pi/2, \quad (4)$$

$$N\phi_c(f_2) = 3\pi/2, \quad (5)$$

respectively, where

$$f_2 = \alpha f_1, \quad (6)$$

$\alpha$  is not necessarily an integer. From (1)-(3), (4) and (5) can be written into the following simpler approximate expression.

$$Pf_1 - Q/f_1 \approx \pi/2, \quad (7)$$

$$Pf_2 - Q/f_2 \approx 3\pi/2, \quad (8)$$

$$P = 2\pi N\sqrt{L_R C_R}, \quad Q = N/(2\pi\sqrt{L_L C_L}), \quad (9)$$

### IV. IMPLEMENTATION

#### A. Artificial CRLH-TL

A schematic of the LH-TL section is shown in Fig. 3. It consists of two unit-cells including two series capacitors of value  $2C$  and one shunt inductor of value  $L$  for symmetry. The RH-TL is a simple microstrip line on each side of the LH section.

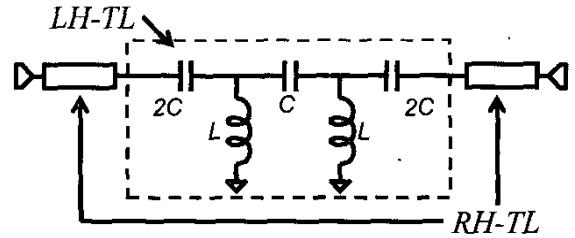


Fig. 3. Schematic of the CRLH-TL used for each branch-line of the proposed design.

The size of this circuit may be reduced by replacing the microstrip line with lumped-distributed-elements [11].

#### B. Procedure of Implementation

From the analysis in section II, the procedure of implementation is:

1. Choose  $f_1$  and  $f_2$
2. Solve (6) to (8) for  $P$  and  $Q$
3. Use  $Q$  to determine the  $L_L C_L$  product with the chosen  $N$
4. Calculate the values of  $L_L$  and  $C_L$  so that  $L_L C_L$  satisfies (9), and (3) is satisfied for  $35\ \Omega$  and  $50\ \Omega$
5. Use  $Pf_1$  or  $Pf_2$  to obtain the electrical length of the RH-TL and hence its physical length using standard microstrip line formulas.

### V. SIMULATION AND MEASUREMENT

Following the procedure in section IV, a practical implementation of the BCL is demonstrated. The operating frequencies of two pass-bands are chosen as  $f_1=930$  MHz and  $f_2=1780$  MHz. The simulation result, including full-wave simulation of the distributed parts, is shown in Fig. 5.

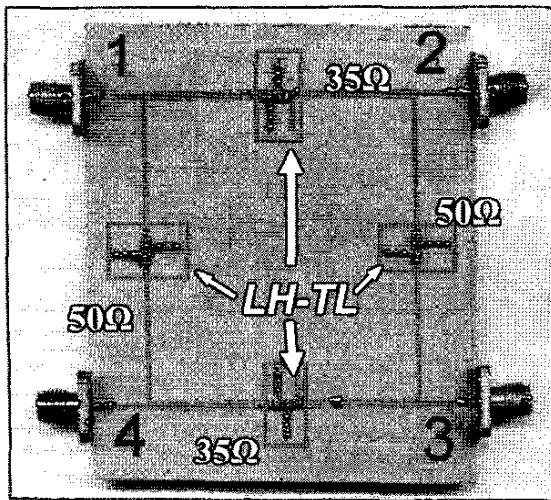


Fig. 4. Photograph of the prototype.

Surface mount chip components provided by Murata Manufacturing Co.,Ltd. were used in the design. The frequency dependence of the of these chip components causes variations of the characteristic impedance of the LH-TL, which results in amplitude imbalance between the two output ports. To compensate for these effects, a tuning stub is added to the  $35\Omega$  CRLH-TLs. The measurement result is shown in Fig.6. The operating frequencies are shifted to 920 MHz at the first pass-band and 1740 MHz at the second pass-band, respectively. In both cases, the phase difference between  $S_{31}$  and  $S_{21}$  is  $\pm 90^\circ$  at  $f_1$  and  $f_2$ , as shown in Fig. 7. The performances in both pass-bands are summarized in Tables I and II, respectively. The 1dB-bandwidth is defined as the frequency range in which the amplitude unbalance between the two output signals is less than 1dB and isolation/return loss is less than  $-10\text{dB}$ .

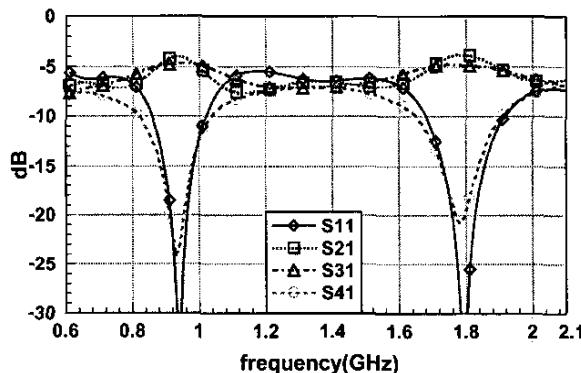


Fig.5. Simulation result for the BLC of Fig. 4.

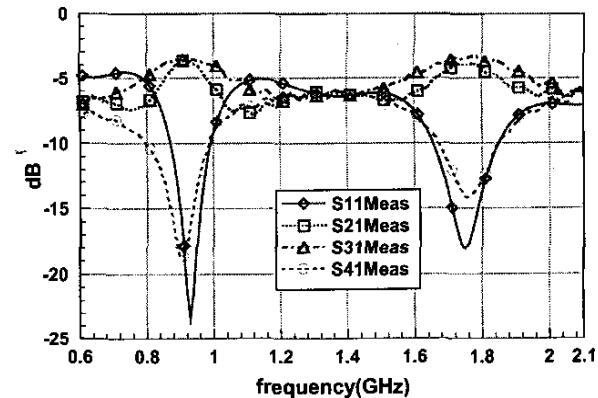


Fig. 6. Measurement result for the BLC of Fig. 4.

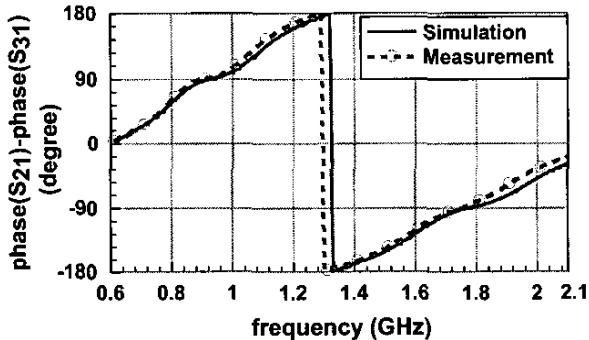


Fig. 7. Phase difference between  $S_{21}$  and  $S_{31}$  of Fig. 4.

TABLE I  
PERFORMANCES IN THE FIRST PASS-BAND

|                     | Simulation    | Measurement   |
|---------------------|---------------|---------------|
| Operating Frequency | 930 MHz       | 920 MHz       |
| Return Loss         | -28.180dB     | -21.242dB     |
| Output 1            | -4.028dB      | -3.681dB      |
| Output 2            | -4.717dB      | -3.593dB      |
| 1dB-Bandwidth       | 140 MHz (15%) | 110 MHz (12%) |
| Isolation           | -24.096dB     | -17.617dB     |
| Phase Difference    | 90.42°        | 91.42°        |

TABLE II  
PERFORMANCES IN THE SECOND PASS-BAND

|                     | Simulation     | Measurement    |
|---------------------|----------------|----------------|
| Operating Frequency | 1780 MHz       | 1740 MHz       |
| Return Loss         | -28.431dB      | -17.884dB      |
| Output 1            | -3.821dB       | -4.034dB       |
| Output 2            | -4.804dB       | -3.556dB       |
| 1dB-Bandwidth       | 100 MHz (5.6%) | 150 MHz (8.6%) |
| Isolation           | -20.821dB      | -13.796dB      |
| Phase Difference    | -89.26°        | -90.96°        |

## VI CONCLUSION

A novel BLC with two arbitrary operating frequencies was proposed. This arbitrariness is obtained by replacing the conventional branch-lines by CRLH-TLs, in which the LH-TL provides an offset from DC and the RH-TL sets the appropriate slope to intercept the two frequencies.

The operation frequencies of the prototype were limited by the self-resonance frequency of the surface mount (SMT) chip components. MMIC implementations of the proposed BLC, overcoming the frequency limitation of SMT chips, may be useful in many dual-band applications of modern mobile communication and WLAN standards.

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